

# A Lab Based Approach in Mathematics

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*"It was the first time I remember  
genuinely enjoying math."*

— MATH 255 student



# Geoff Krall

Author of *Necessary Conditions: How to Teach Secondary Math with Academic Safety, Quality Tasks, and Effective Facilitation*

Assistant Professor at Colorado State University

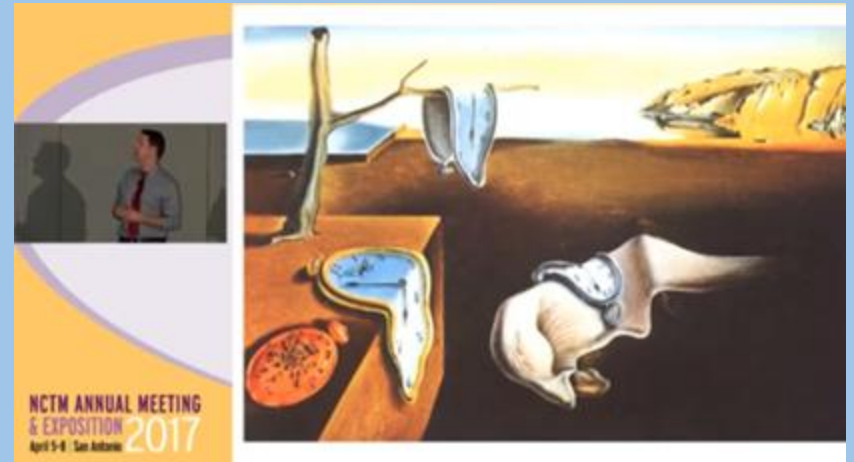


# Dissertation, Scholarship, and Speaking

Krall, G. Teacher-Designed Mathematical Portfolio Assessments: Motivations, Potential Benefits, and Lessons Learned. *Can. J. Sci. Math. Techn. Educ.* 23, 303–321 (2023). <https://doi.org/10.1007/s42330-023-00280-3>

Krall, Geoff. The experience of students engaging in mathematical portfolios: Contributions to mathematical identity and metacognition. University of Wyoming, 2024.

NCTM Shadowcon 2017: “The Art of Mathematical Anthropology”



# MATH 155: Calculus 1 for Biological Scientists

## Course Instructor & Coordinator

- ▶ ~400 students/semester across 8–10 sections (taught by Graduate TAs)
- ▶ Non-negotiable pre-arranged exam dates for the entire course
- ▶ Six quizzes/semester
- ▶ My sections: 50–130 students

→ *Alternative assessment was not really possible*

# MATH 255: Calculus 2 for Biological Scientists

Fall 2025 — Flying Solo

- ▶ Only one section — no coordination constraints
- ▶ No pre-arranged exam or quiz dates
- ▶ Terminal math course for all students
- ▶ Highly specialized cohort of biology students



Discuss

*What would you do if you had nearly total agency in your math class?*

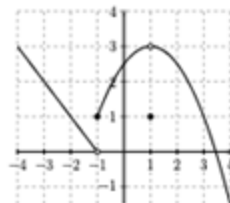
## MATH 155 Quiz 3: §2.1 - §2.7



NAME: \_\_\_\_\_

Student Number:

1. (4 points) For the first question, refer to the following graph of  $f(x)$ . Write "DNE" if a value does not exist.



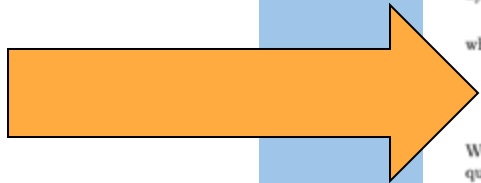
(a)  $f(1) =$

(c)  $\lim_{x \rightarrow 1} f(x) =$

(b)  $\lim_{x \rightarrow 1} f(x) =$

(d)  $\lim_{x \rightarrow 1^-} f(x) =$

2. (4 points) Use the *limit definition of derivatives* to demonstrate that the derivative of  $f(x) = x^2$  is  $f'(x) = 2x$ . You must use the limit definition of a derivative. Credit will not be given if you use derivative rules here. (Note:  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ )



## Lab 1: Integration and Derivatives in Biological Contexts

### Introduction

This lab applies the concepts of derivatives and anti-derivatives to investigate scenarios.

### 1 Rates of Change in a Biological System

In a physiology experiment, the concentration of a hormone in the bloodstream is modeled by the function

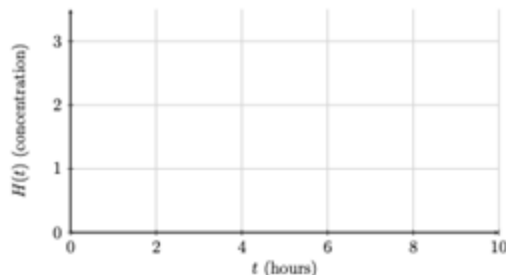
$$H(t) = 4te^{-0.5t}$$

where:

- $H(t)$  is the hormone concentration (in arbitrary units),
- $t$  is time in hours after the hormone is released.

We see this structure of function quite a bit in Bio/Chem. This type of function models a quantity that initially increases due to production or absorption, but later decreases due to natural decay or elimination. It shows exponential growth followed by exponential decay.

1. Sketch a graph of  $H(t)$  for  $t \geq 0$ . Be sure to indicate where the function appears to be increasing and decreasing.



## Course Calendar (Fall 2025)

Week	Monday	Tuesday	Wednesday	Friday
1 (8/25)	0.1 Functions and Trig Review	0.2 Derivative Review	0.3 Integral Review	Lab 1 - Numerical Integration and Derivatives
2 (9/1)	Labor Day	0.4 Advanced Integration Techniques (Substitution)	0.5 Advanced Integration Techniques (Parts) HW1 due	0.6 Advanced Integration Techniques (Alternative Methods)
3 (9/8)	1.1 Using Definite Integrals to Find Area	1.2 Using Definite Integrals to Find Length	Lab 2 - Area Between Curves and Arc Length HW2 due	Lab 2 - Area Between Curves and Arc Length
4 (9/15)	1.3 Using Definite Integrals to Find Volume (Disk Method)	1.4 Using Definite Integrals to Find Volume (Washer Method)	1.5 Density and Center of Mass HW3 due	1.6 Center of Mass (2D)
5 (9/22)	1.7 Improper Integrals	1.7 Improper Integrals	Flex Day HW4 due	Lab 3 - Volume, density, and Center of mass
6 (9/29)	2.1 An Introduction to Differential Equations	2.2 Slope Fields	Review HW5 due	Mid Term Exam 1 (0.1-1.7)

Week	Monday	Tuesday	Wednesday	Friday
7 (10/6)	2.3 Separable differential equations	2.3 Separable differential equations (cont)	2.4 Modeling with differential equations	2.5 Population Growth and the Logistic Equation
8 (10/13)	Population Growth and the Logistic Equation	3.1 Intro to Multivariable Functions	3.2 Partial Derivatives HW6 due	Lab 4 - Differential Equations in Biological
9 (10/20)	3.3 Higher order Partial Derivatives	3.4 The Chain Rule for Partial Derivatives	Partials Consolidation Day HW7 due	4.1 Double Riemann Sums
10 (10/27)	4.2 Double Integrals in a Rectangular Region	4.3 Double Integral over General Regions	4.3 Double Integral over General Regions HW8 due	Lab 5 - Double Integrals
11 (11/3)	5.1 Polar Coordinates	5.2 Integrating with Polar Coords	Review HW9 due	Mid Term Exam 2 (2.1-4.3)
12 (11/10)	5.3 Spherical and Cylindrical Coordinates	6.1 Sequences	6.2 Geometric Series	Lab 6 - Cylindrical Coordinates in Biomedical Structures
13 (11/17)	6.3 Infinite Series	6.4 Alternating Series	6.5 Taylor Series HW10 due	Lab 7 - Taylor Series Approximations
14 (11/24)	Fall Break			

Week	Monday	Tuesday	Wednesday	Friday
15 (12/1)	7.1 Intro to Matrices	7.2 Matrix Operations	7.3 Matrices and Determinants	7.4 Matrices and Eigenvalues and Vectors
16 (12/8)	Lab 8 - Matrices	Review	Final Exam (Part 1) HW11 due	Final Exam (Part 2)
Finals Week				

LAB

# The Lung Model & Cobwebbing

This is a revised version of a lab for a BioCalc class covering recursive functions and a stability analysis technique called “cobwebbing” in the context of a model of a lung.

Teach time unit represents one breath.

## As you work, notice...

### The entry point

Who could get started immediately, regardless of computation skill?

### Meaning vs. procedure

Where does the math feel meaningful rather than just mechanical?

### Adaptation potential

What would you need to change to use this in your own course?

*Work with the people around you — this is meant to be collaborative.*

# One breath — step by step

State at time  $t$

current concentration

$C_t$

mmol/L

Exhale

$p \cdot C_t$  exits

$(1-p) \cdot C_t$

retained

Inhale

$p \cdot \beta$  enters

$+p \cdot \beta$

outside air in

State at time  $t+1$

$C_{t+1}$

$= (1-p)C_t + p \cdot \beta$

mmol/L

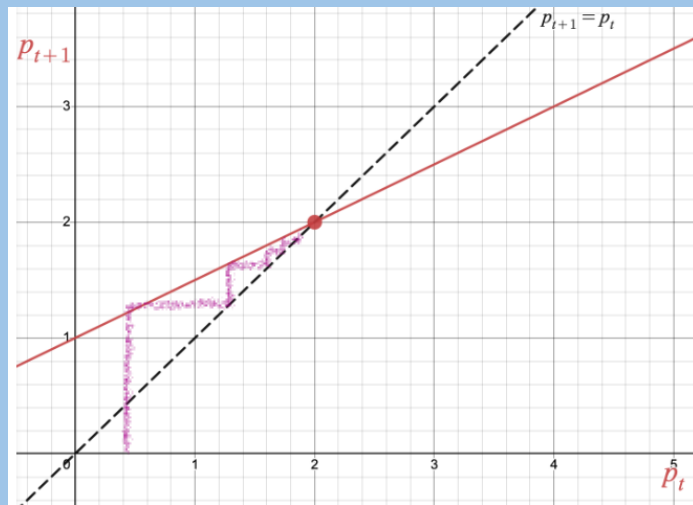
$$C_{t+1} = (1-p) \cdot C_t + p \cdot \beta$$

# Cobwebbing: a method for qualitative behavior in a updating function relationship

Plot the equilibrium line  $p_{t+1} = p_t$  (basically just  $y = x$ )

Plot the updating relationship (basically just  $y = mx + b$  or other form)

- Start at the initial value  $p_0$  ( $p_0 = 0.5$  in the example below), draw a line vertically (purple below) until you hit the updating relationship (red below)
- Move horizontally (left or right) to the equilibrium line (dashed below)
- Vertically (up or down) to the updating relationship
- Repeat



LAB

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## What did you notice?

1

### **About the entry point**

Who could get started immediately? Who might have struggled?

---

2

### **About meaning vs. procedure**

Where did the math feel meaningful rather than just mechanical?

---

3

### **About your context**

What would you need to adapt this for your own course or content?

# Student Feedback

## On Real-World Application

“I think my favorite lab was the cylindrical coordinate DNA helices lab because I could directly apply the calculus knowledge to something I study every day in my classes.”

## On Lower-Stakes Learning



*“I really liked the labs. I felt like they allowed me to apply what I have learned in a **lower stress environment** than a quiz.”*

*“I think this system is **a lot more effective and engaging than testing**, so keep the lab format!”*

## On Understanding the 'Why'



*“Labs helped to solidify the “why” behind it which I always find very beneficial. Lots of the things we do feel very overcomplicated and it's hard to understand why we learn what we do.”*

**Making labs work in  
your context**

# How to Generate Labs for Your Classroom

1

## Identify crucial content

- *Either readily applicable content OR highly important content*

2

## Seek application ideas

- *Optional: survey student interests first (sports, music, biology, environmental science...)*

3

## Create the labs

*I do it in Latex (.tex) to create that professional looking polish*

4

**Swap out 4–8 quizzes with labs**

# Seeking application ideas



... and now for some Fermi estimation.

$$\begin{array}{c}
 \text{9-12 chapters /} \\
 \text{textbook}
 \end{array}
 \times
 \begin{array}{c}
 \text{6-10 sections /} \\
 \text{chapter}
 \end{array}
 \times
 \begin{array}{c}
 \text{25-50 practice} \\
 \text{problems /} \\
 \text{section}
 \end{array}
 =
 \begin{array}{c}
 \text{1350 to 4800} \\
 \text{problems /} \\
 \text{textbook}
 \end{array}$$

● Awful    ● Don't totally suck



13.5 to 48 problems

# PROBLEM BASED MATH CURRICULUM MAPS

UNIT 7.1: Rational Numbers	7-NS.1,2,3	20
		1
		3
		1
		1
		2
		1*
		2
<a href="#">Walking the Line Poster Problem (SERP)</a>	7-NS.2.a	2
<a href="#">A Day Out (MARS)</a>	7-NS.1,2,3	2
<a href="#">Taxi Cabs (MARS)</a>	7-NS.1,2,3	2
<a href="#">Insane Asylum Game (Nora)</a>	7.NS.A.2.B	2
[remediation, extension, assessment, skill practice]		3

[www.gpsa.com.au](http://www.gpsa.com.au)

To the students of Slow Tech High

Throughout the ecological history of Texas, the plight of the Desert Bighorn Sheep has been publicized and, we think, partially solved. However, we need you to evaluate whether or not the species has become robust enough to allow its hunting again.

Among the many species that have been reintroduced at our base or nearby is the desert bighorn sheep. The desert bighorn sheep is important to preserve because it is sensitive to human-related problems in the environment and it is a good indicator of land health.

We have data on the Desert Bighorn going back to the 1800's. We suspect that the population had significantly decreased due to the issue of overhunting in 1950. The source linked below for the Desert Bighorn.

**Effects to Repopulate the Species**

Efforts to reintroduce desert bighorn sheep in Texas began around 1917. By 1955, there were about 400 desert bighorn sheep roaming free in its captivity.

**Timing of Hunting Licenses**

Authorities are beginning to issue licenses to hunt Desert Bighorn in Texas (see article [here](#)). It is your task to write an article measuring the data from desert bighorn in Texas, comparing them to other species that have undergone successful repopulation efforts, and offering suggestions relative to when it is best to issue hunting licenses without the threat of overhunting for the Desert Bighorn.

What we would like you to do is analyze the existing data (looking up to 1957 and the data after 1957). This analysis is critical in evaluating the effects of repopulating the species in Texas.

In the end, we would like you to write an article for *Slow Tech High* and *Wildlife Magazine*, detailing both your analysis and your conclusions as to the timing of hunting permits.

Sincerely,  
 Slow Tech High and Wildlife Magazine

# Seeking application ideas

 Welcome, Geoff

What are some applications for eigenvalues to Calculus 2 for Biological Sciences?

+

Sonnet 4.6 Low



## Stability Analysis of Equilibria

In a nonlinear system, linearization near an equilibrium produces a Jacobian matrix. The signs of the eigenvalues of that Jacobian determine whether the equilibrium is stable, unstable, or a saddle — this is the multivariable analogue of the second-derivative test, and it's highly relevant to biological models (disease-free equilibria in epidemiology, coexistence equilibria in competition models).

Google

applications of eigenvalues in biological sciences

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KITP Online

<https://online.kitp.ucsb.edu> > online > nelson > pdf PDF

### Eigenvectors and eigenvalues in biology

> An intricate eigenvalue spectrum controls the spontaneous activity and induced response. Directed rings of neurons lead to a hole centered on the origin in ... [Read more](#)  
37 pages

Missing: sciences | Show results with: sciences

### Scholarly articles for applications of eigenvalues in biological sciences

... eigenanalysis, with applications to organismal biology - Bookstein - Cited by 55

Networks in life: Scaling properties and eigenvalue ... - Farkas - Cited by 141

Estimating eigenvalues of dynamical systems from time ... - Petrie - Cited by 25



AIP Publishing

<https://pubs.aip.org> > aip > aip > doi > 10.1063/1.4940012\_1\_online

### Applications in Epidemic and Other Bio-inspired Models

2017 · Cited by 5 — Many authors use eigen values to identify either a disease could actually die out in the long-term. This is a typical epidemiological problem ... [Read more](#)

# Components of a Good Lab

## 1) A “Hook”

High-leverage, accessible application that draws students in from the start

## 2) Group-Worthy

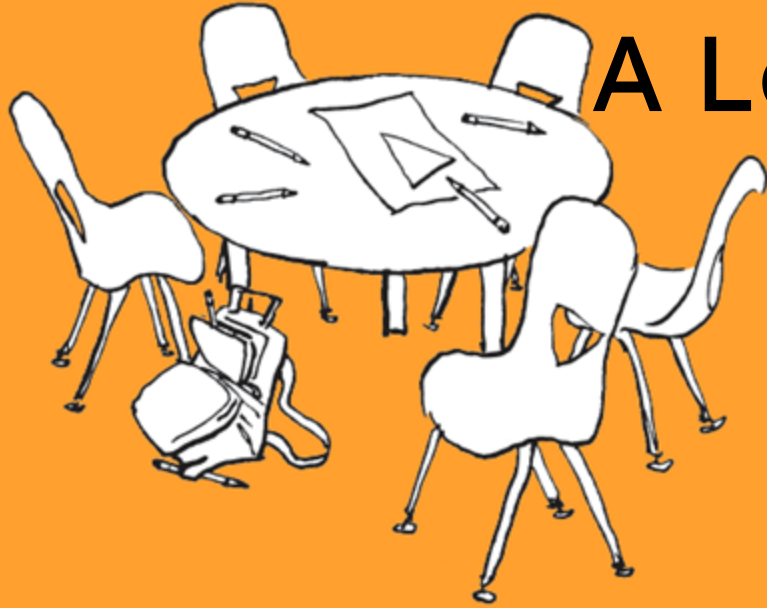
Rich enough that students benefit from collaborating and discussing together

## 3) Interpretation & Analysis

Students make sense of results, not just compute answers

## 4) Reflection Prompts

Metacognitive questions that connect math to the bigger picture



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