Lab: The Lung Model

Content for this lab derived from *Calculus for Biological Scientists*[a]

Setup. With each breath, a person's lungs (volume $V = 6$ L) inhale a volume $W = 3$ L of outside air, replacing half the lung's contents. Let $p = W/V$ be the fraction exchanged, and let β denote the concentration of a gas in outside air. If c_t is the gas concentration in the lungs (mmol/L) after the t-th breath, then:

$$c_{t+1} = (1 - p)\, c_t + p\beta$$

We track a patient with an elevated gas concentration of $c_0 = 6$ mmol/L, and outside concentration $\beta = 3$ mmol/L.

[a] https://jshriner6.github.io/bio-calc1/sec-1-9.html

Part 1: Iterate

Using the recurrence above, identify the updating function for this patient's situation. Then complete the table.

Updating function:

$c_{t+1} =$ _____

t (breaths)	c_t (mmol/L)	c_{t+1} (mmol/L)
0	6.0000	
1		
2		
3		

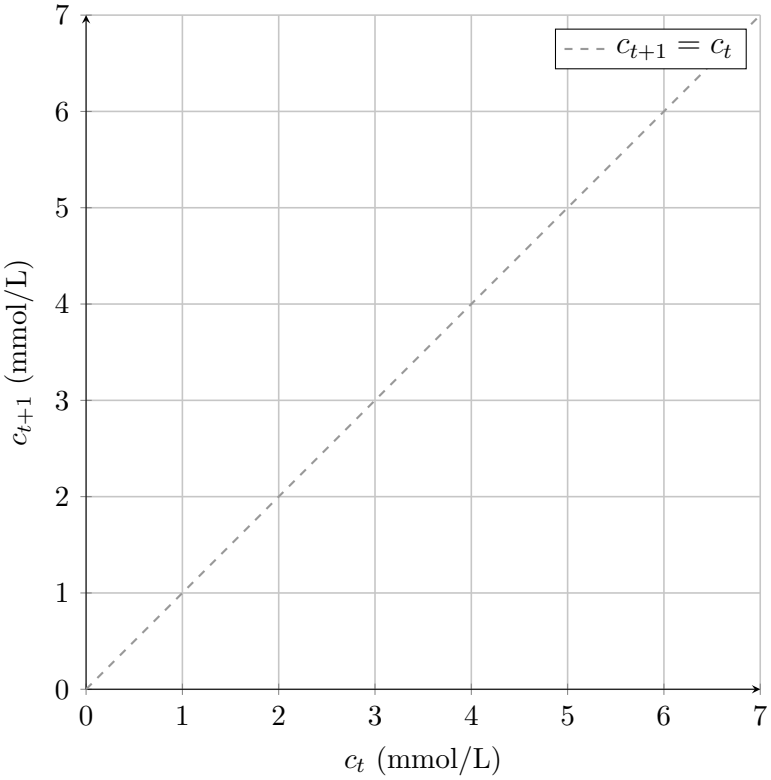
Part 2: Find the Equilibrium

An equilibrium will satisfy the relationship $c_{t+1} = c_t$. From your updating recursive function above, set both c_{t+1} and c_t equal to c^* and solve for c^*. This is your equilibrium concentration.

$c^* =$ _____ mmol/L *How does c^* relate to β? Why does this make sense?*

Part 3: Cobweb Diagram and Stability Analysis

On the axes below, sketch your updating function (treat it just like a slope intercept equation). $c_{t+1} = c_t$ is provided for you. Mark the equilibrium point c^* and the starting value $c_0 = 6$. Then trace the cobweb: go **vertical** to the curve, **horizontal** to the diagonal, and repeat for at least 4 iterations.



Part 4: Interpret

Discussion questions:

1. Your cobweb converges to c^*. Is this equilibrium **stable** or **unstable**? How can you tell from the diagram?

2. A patient with a respiratory condition breathes more shallowly, exchanging only $W = 1$ L per breath (so $p = \frac{1}{6}$). *Without* solving, predict: will their concentration still reach the same c^*? Will it get there faster or slower? What changes in the cobweb diagram?